

Shining on an Orbifold

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Abstract

By shining a hypermultiplet from one side of the bulk of a flat five-dimensional orbifold, supersymmetry can be broken by boundary conditions. The extra dimension is stabilized in a supersymmetric way, and by computing the four-dimensional effective potential for the radion it is shown that supersymmetry breaking does not damage our radius stabilization mechanism. The low energy theory contains the radion and two complex scalars that are massless in the global supersymmetric limit and are stabilized by tree level supergravity effects. It is shown that radion mediation can play the dominant role in communicating supersymmetry breaking to the visible sector. It is also shown that at tree level, contact terms are exponentially suppressed.

I. INTRODUCTION

Supersymmetry (SUSY) and extra dimensions are some of the most active areas of research in high energy physics today. In addition to their mathematically aesthetic value, they might be able to solve the hierarchy problems of particle physics, and both are motivated by string theory. However, the world we live in is four dimensional and not supersymmetric. Therefore if SUSY exists it must be broken, probably spontaneously. And if extra dimensions exist they must be compactified or in some way hidden. These two constraints provide a wealth of possible phenomenology; see for example [1].

Extra dimensions have another problem. If you naively try to compactify them, they are inherently unstable due to Casimir forces. Therefore any self-consistent model with extra dimensions must include a way to stabilize the dimensions against these quantum fluctuations.

One method of doing just that is known as the Goldberger-Wise (GW) mechanism [2]. This was originally designed to stabilize the extra dimension of the RS1 Model [3]. Goldberger and Wise proposed including a scalar field that lived in the bulk but that had independent potentials localized on branes at the two orbifold fixed points. These independent potentials generate a profile for the scalar, and matching boundary conditions enforces a stabilized extra dimension.

A similar idea that involves supersymmetry was considered in [4]. In this paper, the extra dimension is a circle and a hypermultiplet has a source term on a brane located at $y = 0$. This hypermultiplet has an exponential profile in the bulk. Then a “probe brane” is included that interacts with the hypermultiplet. The F-flatness conditions conspire to stabilize the radius of the extra dimension by fitting boundary conditions. This method can also be used to break supersymmetry by fixing the model so that it is impossible to satisfy the F-flatness conditions and the boundary conditions at the same time. Breaking SUSY in this way is generally called “shining” [4, 5].

This paper extends this idea to a flat orbifold. A single hypermultiplet lives in the bulk, and it has sources on branes located at both orbifold fixed points. Fitting boundary conditions overconstrains the problem and forces the radius to be stabilized. A very nice side effect of this model is that supersymmetry need not be broken in order to stabilize the radius. Once we stabilize the radius of the extra dimension we can break supersymmetry using the same technique. We shine another hypermultiplet from the brane at $y = 0$ and find that we cannot match boundary conditions and preserve supersymmetry at the same time. We show that that this SUSY breaking does not have any sizeable effect on the radius stabilization mechanism. This method is improved from [4] since the orbifold geometry means that we do not need any chiral superfields living on one of the branes.

Our model is similar to one proposed previously by Maru and Okada, but they consider the warped case [6]. However, they claim that there is no viable flat space limit. We show why this is not correct. We will also correct a claim about the zero modes of the 4D effective theory.

In the next section we will present the model and show how the shining mechanism can be used to both stabilize the radius and break supersymmetry. In the following section we will consider the four-dimensional effective theory that reproduces the low energy physics. We will also discuss how supergravity effects help stabilize the flat directions, and how radion [7] and anomaly [8] mediated supersymmetry breaking can occur.

II. THE MODEL

In this section we will present the model in terms of $N = 1$ superfields in five dimensions. We work with a single extra dimension compactified on a flat orbifold S^1/\mathbb{Z}_2 :

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu - R^2 dy^2 \quad (1)$$

where we are using a mostly minus metric throughout this paper. R is the radius modulus field, or “radion”, which parameterizes the size of the extra dimension, and $y \in [0, \pi]$ is an angular variable. The orbifold parity defines a symmetry under the transformation $y \rightarrow -y$. The five-dimensional fields in the theory will be either even or odd under this parity.

This model consists of two hypermultiplets (Φ, Φ^c) and (Ψ, Ψ^c) that are shined across the bulk from a brane located at $y = 0$. One of these hypermultiplets will be used to stabilize the extra dimension while the other one will be used to break supersymmetry. In the convention that we use, the conjugate superfields are even under the orbifold parity while the other chiral superfields are odd.

The five-dimensional action for our model is given by [9, 10]:

$$\begin{aligned} S = & \int d^4x dy \int d^4\theta \varphi^\dagger \varphi \frac{T + T^\dagger}{2} \{ -3M_5^3 + \Phi^\dagger \Phi + \Phi^{c\dagger} \Phi^c + \Psi^\dagger \Psi + \Psi^{c\dagger} \Psi^c \} \\ & + \int d^4x dy \int d^2\theta \varphi^3 \{ \Phi^c (\partial_y + mT) \Phi + \Psi^c (\partial_y + \mu T) \Psi \} + \text{h.c.} \\ & + \int d^4x dy \int d^2\theta \varphi^3 \left\{ \Phi^c [J\delta(y) - J' \delta(y - \pi)] + \Psi^c K\delta(y) + \alpha\delta(y) \right\} + \text{h.c.} \end{aligned} \quad (2)$$

where φ is the conformal compensator and T is the radion superfield¹ (see Appendix A). α is a constant superpotential living on the $y = 0$ brane that will be used to cancel the cosmological constant after SUSY breaking. Notice that this action has a $U(1)_R$ symmetry in the bulk and the $y = \pi$ brane with $R(\Psi^c) = R(\Phi^c) = +2$ and all other superfields neutral. This symmetry is explicitly broken on the $y = 0$ brane by the α term. This will be important later. Also notice that if we extend our domain in y to the covering space $y \in [-\pi, \pi]$ the mass terms contain a sign function. We leave this out to avoid the cumbersome notation, but it is very important when going to the four dimensional effective theory.

This model is virtually identical to the model of Maru and Okada [6]. In that paper the authors stabilized the extra dimension in the case of a warped background using a hypermultiplet with delta-function sources on both branes. However they claim that the only way this can be done is in warped space and that if you take the flat space limit you get a runaway potential for the radion. This is not the case if you take the appropriate flat space limit. Specifically, they parameterized their bulk masses in terms of a c -parameter: $m = (\frac{3}{2} + c)k$ where k is the curvature in the warp factor. Then if you naively take the limit $k \rightarrow 0$ the bulk masses would vanish and the radion would no longer be stabilized. The appropriate thing to do is to take the limit as $k \rightarrow 0$ while holding the bulk mass fixed. It

¹ Notice the T dependence in the bulk mass term for the hypermultiplet. This dependence was not included in Equations 11-14 of [9]. However their later inclusion of F_T in the action was correct, so this does not change any of their results. Therefore we assume that this is simply a typo in their paper.

is easy to take this limit in their paper and we get the same results presented here for the radion potential.

As a first step in analyzing the model we ignore supergravity contributions, so $T = R$ and $\varphi = 1$; in other words, $F_T = F_\varphi = 0$. We will come back to this in a later section. With these conditions the remaining F-term equations of motion are:

$$RF_\Phi^c = (mR + \partial_y)\phi + [J\delta(y) - J'\delta(y - \pi)] \quad (3)$$

$$RF_\Psi^c = (\mu R + \partial_y)\psi + K\delta(y) \quad (4)$$

$$RF_\Phi = (mR - \partial_y)\phi^c \quad (5)$$

$$RF_\Psi = (\mu R - \partial_y)\psi^c \quad (6)$$

Supersymmetry is maintained if we can find (y -dependent) vevs of the scalar fields so that all of the above F-terms vanish. Let us first consider the F-flatness condition $F_\Phi^c = 0$. The first delta function gives the boundary condition $\phi(0) = -\frac{J}{2}$ so there is a unique solution:

$$\phi(y) = -\frac{J}{2}\Theta(y)e^{-mR|y|} \quad (7)$$

where $\Theta(y)$ is the Heaviside step function with the convention $\Theta(0\pm) = \Theta(\pi\mp) = \pm 1$. The boundary condition at $y = \pi$ then overconstrains the problem and fixes the radius:

$$R = \frac{1}{m\pi} \log\left(\frac{J}{J'}\right) \quad (8)$$

Hence this model stabilizes the size of the extra dimension as long as $|J| > |J'|$ and they each have the same sign.

The Ψ sector breaks supersymmetry through the shining mechanism [4]. To understand how this works notice that if we set $F_\Psi^c = 0$ we can write down the solution:

$$\psi(y) = -\frac{K}{2}\Theta(y)e^{-\mu R|y|} \quad (9)$$

The coefficient is set by the delta function source on the $y = 0$ brane. Notice however that there is no source on the $y = \pi$ brane; combined with the fact that $\psi(y)$ is an odd field the boundary condition is $\psi(\pi) = 0$. This boundary condition is inconsistent with Equation (9), and therefore supersymmetry is broken on the boundary at $y = \pi$.

Finally let us consider the last two F-terms. Setting these equations to zero gives the general results:

$$\phi^c = Be^{mR|y|} \quad (10)$$

$$\psi^c = Ce^{\mu R|y|} \quad (11)$$

The coefficients B and C are arbitrary and represent an indetermination of the four-dimensional zero modes of these scalars. Hence, upon integrating out the fifth dimension these fields correspond to flat directions.

That there are two flat directions in our theory should come as no surprise [11]. ψ^c is the scalar field in the multiplet that breaks supersymmetry ($F_\Psi^c \neq 0$), so it is expected to be flat at tree level. That ϕ^c is also a flat direction should not surprise us either. It is due to

the fact that the condition $F_\Phi^c = 0$ was used to stabilize the extra dimension, i.e.: give the radion a mass. This leaves over an extra degree of freedom corresponding to the massless ϕ^c . This interpretation of the flat directions differs from [6]; this difference will be clarified when we discuss the 4D effective theory.

III. 4D SPECTRUM

Now we will consider the four-dimensional effective theory generated by the action in Equation (2). In the first section we will derive the effective potential for the radion and SUSY breaking by setting all the hyper-scalars to their vevs from the previous section. In the next section we will consider the contributions coming from the hyper-scalars and write down an effective superpotential and Kahler potential that captures these effects. In the third section we will consider the lowest order effects of supergravity (turning F_φ and F_T back on). In the final section we will look at how other fields are affected by the shining field. We consider the specific examples of putting matter on one of the branes, and of putting a gauge field in the bulk.

A. Radion Potential

We now wish to construct an effective potential for the radion. In the process we will also be able to parameterize the size of supersymmetry breaking. In order to do this we need to compute the four-dimensional effective potential. Ignoring any contributions from supergravity this potential is given by:

$$V = \int_0^\pi dy R [|F_\Psi|^2 + |F_\Psi^c|^2 + |F_\Phi|^2 + |F_\Phi^c|^2] \quad (12)$$

There is a very nice way to understand Equation (12) that was presented in [4]: think of the extra dimension coordinate y as a (continuous) index for the chiral superfields. Then the potential is nothing more than the sum of all of the magnitude-squared F-terms, which is precisely what Equation (12) is. F_Ψ and F_Φ are proportional to the flat directions so they will not contribute to the effective potential at tree level. We will see how the zero modes of the even scalars contribute to the effective potential in a later section. This leaves two terms to calculate.

Supersymmetry is explicitly broken in the F_Ψ^c term. To isolate that result we must consider the full equations of motion for the scalar field upon integrating out the auxiliary fields. Rather than do that explicitly, we employ the following trick, which is equivalent. We insist that the boundary conditions on the fields are sacred; therefore $\psi(\pi) = 0$ must be enforced. We have already seen that this condition cannot be satisfied for $F_\Psi^c = 0$ but we can get as close as possible if we make the following ansatz:

$$\psi(y) = -\frac{K}{2}\Theta(y) [e^{-\mu R|y|} - e^{-\mu R\pi}f(y)] \quad (13)$$

where $f(y)$ is some function that satisfies the boundary conditions $f(0) = 0$, $f(\pi) = 1$. This will enforce the boundary condition but at the cost of introducing a term into the potential:

$$\Delta V = \int_0^\pi dy \frac{K^2}{4R} e^{-2\mu R\pi} |\partial f - \mu R f|^2 \quad (14)$$

Now we can chose this function to minimize the potential. Performing this minimization using variational methods and using the boundary conditions gives:

$$f(y) = \frac{\sinh(\mu Ry)}{\sinh(\mu R\pi)} \quad (15)$$

We can plug this result back into Equation (14) and integrate over y to get:

$$\Delta V = \frac{1}{2} \frac{\mu K^2}{e^{2\mu\pi R} - 1} \quad (16)$$

F_Φ^c vanishes only when $R = r_0$, the stabilized radius defined in Equation (8). For an arbitrary radius, $F_\Phi^c \neq 0$ and we can repeat the above steps exactly for $\phi(y)$ appearing in F_Φ^c . We try the ansatz:

$$\phi(y) = -\frac{J}{2}\Theta(y) \left[e^{-mR|y|} - \left(\frac{J'}{J} - e^{-mR\pi} \right) g(y) \right] \quad (17)$$

where $g(y)$ has the same boundary conditions as $f(y)$. Indeed, upon minimizing the potential we find that $g(y)$ has the same form as $f(y)$ with μ replaced by m . Plugging it back into Equation (12) and integrating over y we find:

$$V(R) = \frac{1}{2} \frac{m(J - J' e^{m\pi R})^2}{e^{2m\pi R} - 1} + \Delta V \quad (18)$$

This potential is minimized for the radius given in Equation (8). Near this stabilized radius $\Delta V \sim \mu K^2 (J/J')^{-2\mu/m}$ does not give a significant correction relative to the first term due to the exponential suppression for even moderate values of the parameters. For concreteness, we chose the parameters: $J = K = M_5^{3/2}/10$, $J' = M_5^{3/2}/100$, $\mu = M_5/10$ and $m = M_5/75$. Then we find $R \sim 55l_5$ where l_5 is the 5D Planck length. This generates a compactification scale $M_c \sim 0.02M_5$. Using the well-known relation $M_P^2 = M_5^3/M_c$, we estimate $M_5 \sim 10^{17}$ GeV. We estimate the vacuum energy at this radius to be $M_{SUSY} = 3 \times 10^{-5}M_5 \sim 10^{12}$ GeV.

We can take the second derivative of this potential to find the mass of the radion. After taking into account the normalization of the radion (see Appendix A) we find $m_r \sim 10^{-3}M_P \sim 10^{15}$ GeV for the above values of the parameters.

B. Higher Modes and the Effective Superpotential

To get the effective scalar potential in four dimensions we must do a KK expansion of the fields. The details of this are reviewed in Appendix B. Here we quote the results:

$$\phi(x, y) = -\frac{J}{2}\Theta(y)e^{-mR|y|} + \sqrt{\frac{2}{\pi}} \sum_n \phi_n(x) \sin(ny) \quad (19)$$

$$\phi^c(x, y) = -B(x)e^{+mR|y|} + \sqrt{\frac{2}{\pi}} \sum_n \phi_n^c(x) \sin \left[ny + \tan^{-1} \left(\frac{n}{mR} \right) \right] \quad (20)$$

and similarly for (ψ, ψ^c) with $(B, m, J) \rightarrow (C, \mu, K)$. The KK masses are given by the simple relation: $M_n^2 = m^2 + n^2/R^2$ ($n > 0$) for both ϕ and ϕ^c (ψ and ψ^c) and $M_B = M_C = 0$. The

minus sign in front of $B(x)$ is inserted for later convenience. The first term in Equation (19) is a y -dependent vev. There is no zero mode for the odd field, as explained in Appendix B. This is another correction to [6], who suggest that the zero mode of the odd field corresponds to the flat direction. This role is played by the even zero mode $B(x)$, as explained earlier.

To get the 4D effective theory we insert this result into the full five-dimensional Lagrangian and integrate over y . Since the KK modes all have masses at the compactification scale or higher they should not seriously affect the low energy physics; we will see that they decouple below. We also have the (y -dependent) vev of the odd field; that just gives us the potential previously calculated in Equation (18). We are left with the zero mode for the even field:

$$\mathcal{L}_4 = \int_0^\pi dy e^{2mRy} |\partial B|^2 = \frac{1}{2m} (e^{2mR\pi} - 1) |\partial B|^2 + \mathcal{O}(\partial R) \quad (21)$$

Now define $R = r_0 + r$. We can canonically normalize the field $B(x)$ by making the field redefinition: $B \rightarrow B \left(\frac{2m}{e^{2m\pi r_0} - 1} \right)^{1/2}$ and we finally have (after including the ψ -sector):

$$\begin{aligned} \mathcal{L}_4 = & |\partial B|^2 + \lambda |\partial B|^2 [2\pi m r + 2\pi^2 m^2 r^2 + \dots] \\ & + |\partial C|^2 + \tilde{\lambda} |\partial C|^2 [2\pi \mu r + 2\pi^2 \mu^2 r^2 + \dots] + \mathcal{O}(\partial r) \\ & - V(r_0 + r) \end{aligned} \quad (22)$$

where $V(r_0 + r)$ is the potential in Equation (18) and the terms in brackets come from expanding $2e^{\pi m r} \sinh(\pi m r)$. Using Equation (8):

$$\lambda = \frac{1}{1 - e^{-2m\pi r_0}} = \frac{1}{1 - (J'/J)^2} \quad (23)$$

$$\tilde{\lambda} = \frac{1}{1 - e^{-2\mu\pi r_0}} = \frac{1}{1 - (J'/J)^{2\mu/m}} \quad (24)$$

Equation (22) is the four-dimensional effective Lagrangian for the canonically normalized scalar field zero modes and their lowest order couplings to the radion.

In addition to Equation (22), there are also terms that involve the derivative of R . These terms are already quadratic in the B field, so they represent other higher order effects that do not interest us here.

The higher KK modes do not have any problem or ambiguity in their coupling to the radion, which comes from the KK mass term:

$$\Delta \mathcal{L} = - \sum_n \phi_n^{c\dagger} \left\{ \partial^2 + \left[m^2 + \frac{n^2}{r_0^2} \left(1 + \frac{r}{r_0} \right)^{-2} \right] \right\} \phi_n^c$$

Now we would like to write down the four-dimensional Lagrangian in terms of superfields. The only relevant fields that appear in the low energy theory are the B, C scalars and the

radion. The kinetic terms and the interaction terms can be derived from a Kahler potential²:

$$K_4 = \mathcal{B}^\dagger \mathcal{B} \left(e^{m\pi(T+T^\dagger)} - 1 \right) + \mathcal{C}^\dagger \mathcal{C} \left(e^{\mu\pi(T+T^\dagger)} - 1 \right) \quad (25)$$

where \mathcal{B} and \mathcal{C} are the four dimensional chiral superfields containing B and C respectively. We also need to write down a superpotential that gives us Equation (18):

$$W_4 = -\sqrt{\frac{m}{2}} \left(J - J' e^{m\pi T} \right) \mathcal{B} - \sqrt{\frac{\mu}{2}} K \mathcal{C} \quad (26)$$

This choice for the Kahler potential and superpotential will, after the appropriate canonical rescaling, reproduce Equation (22).

C. Effects from Supergravity

We are now in a position to incorporate effects from supergravity. We start with the effective four-dimensional Lagrangian:

$$\mathcal{L}_4 = \int d^4\theta \varphi^\dagger \varphi \left\{ -\frac{3}{2} M_5^3 (T + T^\dagger) + K_4 \right\} + \int d^2\theta \varphi^3 (W_4 + \alpha) + \text{h.c.} \quad (27)$$

where the first term is the supergravity contribution derived in [12] and K_4 and W_4 are given in Equation (25) and (26) respectively. The constant α is required to cancel the cosmological constant in order to properly normalize the gravitino mass [13]. The details of deriving Equation (27) from the full 5D theory can be found in [14]. The superpotential for \mathcal{C} is reminiscent of the Polonyi model [15]. In Polonyi models, the vev of the scalar field is pushed up to the Planck scale. This will happen here as well, but it does not do any damage to our results [16].

First we integrate out the auxiliary fields to get a scalar potential. After rescaling the fields so they have canonical kinetic terms as in Equation (22), we get:

$$V_4(B, C, R) = V(R) + \frac{2R}{3M_5^3} \left\{ X_{\tilde{B}} [\tilde{B} - \langle \tilde{B} \rangle]^2 + X_{\tilde{C}} [\tilde{C} - \langle \tilde{C} \rangle]^2 \right\} - U_0 + \mathcal{O}(M_5^{-6}) \quad (28)$$

where $V(R)$ is the potential given in Equation (18), $U_0 = \frac{2R}{3M_5^3} (X_{\tilde{B}} \langle \tilde{B} \rangle^2 + X_{\tilde{C}} \langle \tilde{C} \rangle^2)$, and

$$\tilde{B} \equiv \frac{1}{\sqrt{1+\epsilon^2}} (B + \epsilon C) \quad (29)$$

$$\tilde{C} \equiv \frac{1}{\sqrt{1+\epsilon^2}} (C - \epsilon B) \quad (30)$$

So we find that the B and C fields mix, but they can be redefined to have definite masses and vevs. These quantities along with the mixing parameter ϵ are given in Appendix C. If we

² There is a subtlety here. When writing down the Kahler and superpotential we must match to the component Lagrangian before rescaling the fields. So Equations (25) and (26) are actually found from matching to Equation (21) after a field redefinition $B \rightarrow \sqrt{2m}B$, $C \rightarrow \sqrt{2\mu}C$ to get the dimensions right.

remove the C field (no supersymmetry breaking³) but there is still a cosmological constant (so $\alpha \neq 0$) then we find that $\langle B \rangle = \sqrt{\frac{m}{2}} \frac{3\alpha}{m^2 \pi r_0 J'}$. This is exactly as we expect from [14].

All of the above masses and vevs depend on the radius, but we have fixed $R = r_0$, the radius fixed by the Φ -sector given in Equation (8). There is also mixing with the radion, and supergravity will give additional contributions to the radion mass; this is not very important since $V(R)$ generates a radion mass just below the compactification scale while supergravity effects are all suppressed by powers of the Planck scale. So it is sufficient to fix the radion at r_0 since any radion mixing with the scalars will be very small. This means that there are actually two sources of supersymmetry breaking: one source comes from the C field directly ($F_C \neq 0$), and another source from the fact that $R = r_0$ is not the true minimum of the potential in Equation (18). We claim that the second source of supersymmetry breaking is negligible compared to the first. This can be seen by letting $R_{\text{truevac}} = r_0 + \delta$, where δ is small from the argument following Equation (18). In fact, a numerical analysis shows that for the values of parameters given, $\delta \sim 0$ to a very good approximation. Therefore we need not worry about these additional contributions.

The masses of the scalars can be computed for the values of the parameters mentioned below Equation (18). We find $m_{\bar{B}} \sim 10^{12}$ GeV, and $m_{\bar{C}} \sim 10^7$ GeV. Both of these masses are well below the compactification scale and m_r as promised.

Finally, we demand that the cosmological constant be tuned to zero. Fixing the radion to its classical value and the scalar fields to their vevs gives $V(r_0) - U_0 = 0$. This can easily be solved for α ; see Appendix C.

We can now use the formula to compute $\langle F_\phi \rangle$ and $\langle F_T \rangle$. We find⁴:

$$\langle F_\phi^\dagger \rangle = \frac{\alpha}{M_5^3 r_0} - \frac{\sqrt{2\mu} K (J'/J)^{\mu/m} \langle C \rangle}{3M_5^3 r_0} - \frac{\langle F_T^\dagger \rangle}{2r_0} \quad (31)$$

$$\frac{\langle F_T^\dagger \rangle}{2r_0} = \frac{\frac{3\alpha}{2r_0} - \sqrt{\frac{\mu}{2}} \frac{K}{r_0} \left(1 + \mu\pi\tilde{\lambda}r_0\right) (J'/J)^{\mu/m} \langle C \rangle - \sqrt{\frac{m}{2}} m\pi J' \langle B \rangle}{2r_0(1-\lambda)m^2\pi^2\langle B \rangle^2 + 2r_0(1-\tilde{\lambda})\mu^2\pi^2\langle C \rangle^2 + \frac{3}{2}M_5^3} \quad (32)$$

The first term in Equation (31) cancels the cosmological constant; the second term comes from the SUSY-breaking F -term (F_C); the final term is the radion-mediated contribution given in Equation (32). For the given parameters this generates $\frac{|\langle F_T \rangle|}{2r_0} \sim 10^6$ GeV and $m_{3/2} = \langle F_\varphi \rangle \sim 10^6$ GeV. These quantities are the same order of magnitude due to the large Polonyi vev $\langle C \rangle$ which can cancel the cosmological constant term in Equation (31). Notice that in the limit considered earlier where $C \equiv 0$ but there is still a cosmological constant, we find to this order after plugging in our result for $\langle B \rangle$ given below Equation (30) that $\langle F_T^\dagger \rangle = 0$ and $\langle F_\varphi^\dagger \rangle = \frac{\alpha}{M_5^3 r_0}$, again in agreement with [14].

³ This can be thought of as the limit $K \rightarrow 0$ since in that case the Ψ sector would have no odd profile in the bulk.

⁴ Notice that these vevs are of the original fields. They can be computed by inverting Equations (29-30).

D. Soft Masses from the Shining Sector

We now ask what happens to the MSSM in our model of SUSY breaking. In the full 5D theory, supersymmetry is broken near the brane at $y = \pi$. Thus, we can place the MSSM on the brane at $y = 0$ and ask if this will generate any contact interactions in the 4D effective theory. Such terms would look like:

$$\mathcal{L}_c = \int_0^\pi dy \delta(y) \int d^4\theta \frac{Q^\dagger Q \Psi^{c\dagger} \Psi^c}{M_5^3} \quad (33)$$

where Q is a chiral superfield in the MSSM.

Now it is sufficient to only consider the zero mode of the hyper-scalar since all of the KK modes have masses of order the compactification scale or higher, and these will generate Planck and Yukawa suppressed interactions. In this case:

$$\Psi^c(x, y) = \sqrt{\frac{2\mu}{e^{2\mu\pi R} - 1}} \mathcal{C}(x) e^{\mu R|y|} \quad (34)$$

is the canonically normalized mode. This will generate contact terms of the form:

$$\mathcal{L}_c = \int d^4\theta \frac{\mu}{M_5^3} Q^\dagger Q \mathcal{C}^\dagger \mathcal{C} e^{-2\mu\pi r_0} \quad (35)$$

and this gives a contribution to the masses of the MSSM scalars:

$$\Delta m_{\tilde{q}}^2 = \frac{\mu |F_C|^2}{M_5^3} e^{-2\mu\pi r_0} \sim \frac{\mu M_{SUSY}^4}{M_5^3} \left(\frac{J}{J'} \right)^{-2\mu/m} \quad (36)$$

So these contact interactions will be exponentially suppressed at tree level. One could have guessed that this would be the case, since the wavefunction of the zero mode of the even field is an exponentially increasing function of y . Thus the bulk scalar likes to spend all of its time far away from the visible brane at $y = 0$. However, we generally expect that radiative corrections might spoil this result and must be checked in models that incorporate this shining mechanism.

Now consider putting a gauge field in the bulk (for simplicity, let it be a $U(1)$ gauge field, but it does not have to be). This would give an extra contribution to the action:

$$\Delta \mathcal{L}_4 = \int d^2\theta \frac{T}{4g_5^2} \mathcal{W}_\alpha \mathcal{W}^\alpha + \text{h.c.} \quad (37)$$

This term generates a contribution to the gaugino mass through radion mediation [7]:

$$\Delta m_{1/2}^{(RMSB)} = \frac{\langle F_T \rangle}{2r_0} \sim m_{3/2} \quad (38)$$

Anomaly mediation also gives a contribution to the gaugino masses. This formula is complicated somewhat by the fact that the Polonyi model has a Planck-scale vev [17], but the important point is that $\Delta m_{1/2}^{(AMSB)} \ll \Delta m_{1/2}^{(RMSB)}$ due to a loop factor. So radion mediation is the dominant contribution to $m_{1/2}$ coming from supergravity.

We can also have contact interactions between the gauge field and the shining field:

$$\Delta\mathcal{L} = \int_0^\pi dy \int d^2\theta \frac{\Psi^c \mathcal{W}_\alpha \mathcal{W}^\alpha}{M_5^{3/2}} \quad (39)$$

After plugging in Equation (34), this will introduce a new contribution to the gaugino mass:

$$\Delta m_{1/2}^{(C)} = \frac{|F^C|}{M_5} \sim \frac{M_{SUSY}^2}{M_5} \quad (40)$$

Thus this contact term gives a contribution to the gaugino mass $\Delta m_{1/2}^{(C)} \sim 10^7$ GeV, which is comparable to $\Delta m_{1/2}^{(RMSB)}$ at tree level.

We can suppress this contribution to the gaugino mass by making use of the $U(1)_R$ symmetry mentioned below Equation (2). From Equation (37) we see that $R(\mathcal{W}_\alpha) = +1$ so that the contact term in Equation (39) breaks the R symmetry by 2 units. Thus it can only be generated on the $y = 0$ brane where the α term has already broken the R-symmetry. Thus the generated contact term in Equation (39) will come with a delta function.

$$\Delta\mathcal{L} = \int_0^\pi dy \delta(y) \int d^2\theta \frac{\Psi^c \mathcal{W}_\alpha \mathcal{W}^\alpha}{M_5^{3/2}}$$

Plugging in Equation (34) for Ψ^c and integrating over y will now generate an exponentially suppressed contribution to the mass in analogy with Equation (36):

$$m_{1/2}^{(C)} \sim \frac{M_{SUSY}^2}{M_5} \left(\frac{J}{J'} \right)^{-\mu/m} \ll \Delta m_{1/2}^{(RMSB)}$$

So we find that it is possible for the RMSB contribution to dominate the gaugino mass.

IV. DISCUSSION

This paper has extended the shining mechanism of supersymmetry breaking to the geometry of flat orbifolds. This is a very nice way to break supersymmetry via a hidden sector in extra dimensions. It avoids the need for extra superfields living on the boundary branes as in [4]. It can easily be extended to other interesting situations such as matter or gauge fields in the bulk, where radion mediation can play an important role.

This paper has also clarified some of the issues raised in [6]. In particular, contrary to their claim, it *is* possible to fit their model to the flat case and there is nothing special about the warped geometry. We have also clarified the role of the zero modes in the low energy theory.

In addition we have shown how supergravity plays the usual role of radiative corrections in stabilizing the flat scalars. This is because our model is actually a Polonyi model in disguise, which is a free field theory in the limit $M_5 \rightarrow \infty$.

This model of supersymmetry breaking only introduces exponentially suppressed contact terms at tree level when the MSSM is put on the brane at $y = 0$. So it might be possible to generate realistic soft masses for the squarks and sleptons. In addition, radion mediated SUSY breaking might play an important part if the bulk contact terms can be suppressed.

Here, this was accomplished by imposing an R-symmetry that originally appeared as an accidental symmetry in the bulk and is broken on the brane at $y = 0$.

The classic example of a model with radion mediation as the dominant mechanism of SUSY breaking is the “no-scale model” [18], where $F_\varphi \equiv 0$ [9]. This model is known to be unstable after radiative corrections are included. Recently, it has been improved by including a general stabilization mechanism and a constraint was derived to keep the model “almost no-scale” [19]:

$$\langle K_{T+T} \rangle \ll \frac{M_5^3}{2\pi r_0}$$

where K is a radius-stabilizing Kahler potential. This constraint corresponds to making sure that F_φ remains small relative to F_T/r_0 . The model considered here violates this constraint: both sides of the inequality are the same order of magnitude. This is because our model has $F_\varphi \sim F_T/r_0$. Anomaly mediation is then suppressed by a loop factor, not a small F_φ . This is what leads to dominant radion mediation.

Finally, notice that this model, although in flat space, has a Kahler potential that depends on the exponential of the radion. This is reminiscent of warped space, and there might be a corresponding reinterpretation of the effective four-dimensional theory. This could lead to interesting consequences for AdS/CFT, warped supergravity, etc, and is left for future research.

Acknowledgments

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APPENDIX A: THE RADION MODULUS FIELD

The radion modulus field comes from the gravitational part of the action. To see how this comes about consider the usual Einstein-Hilbert action in five dimensions:

$$S_5 = \int d^5x \sqrt{-G} \{ M_5^3 \mathcal{R}_{(5)} + \mathcal{L}_{(5)M} \} \quad (\text{A1})$$

where M_5^3 is the five-dimensional Planck scale, G is the determinant of the five-dimensional metric, $\mathcal{R}_{(5)}$ is the five-dimensional Ricci scalar and $\mathcal{L}_{(5)M}$ contains any other fields. We can work in the gauge (coordinate system) where $G_{5\mu} \equiv 0$ so the differential line element is:

$$ds^2 = G_{MN} dx^M dx^N = g_{\mu\nu}(x, y) dx^\mu dx^\nu - r^2(x) dy^2 \quad (\text{A2})$$

Our convention is that the metric is mostly minus. $g_{\mu\nu}$ is the induced four-dimensional metric which is generally a function of the five-dimensional spacetime, and $G_{55} \equiv -r^2$ is

assumed to be independent of the extra dimension. Then $\sqrt{-G} = \sqrt{-g} \times r$ and upon carefully expanding the Ricci scalar, our action is:

$$S_5 = \int d^5x \sqrt{-g} \left\{ r M_5^3 (\tilde{\mathcal{R}}_{(4)} + \delta\mathcal{R}) + r \mathcal{L}_{(5)M} \right\} \quad (\text{A3})$$

where

$$\mathcal{R}_{(4)} \equiv \int dy \tilde{\mathcal{R}}_{(4)}$$

is the four-dimensional Ricci scalar and $\delta\mathcal{R}[g, r]$ are the terms in the five-dimensional Ricci scalar that depend on the fifth dimension explicitly.

In a flat extra dimension the four-dimensional graviton $g_{\mu\nu}$ is independent of y , so the y -dependence has been completely isolated and we can easily perform the integral over the fifth dimension⁵. However, the graviton kinetic term is no longer canonical. To fix this problem we can do a Weyl rescaling of the metric [20]:

$$\begin{aligned} g_{\mu\nu} &\longrightarrow \Omega^2 g_{\mu\nu} \\ g^{\mu\nu} &\longrightarrow \Omega^{-2} g^{\mu\nu} \end{aligned}$$

Under this transformation:

$$\sqrt{-g} \longrightarrow \Omega^4 \sqrt{-g} \quad (\text{A4})$$

$$\mathcal{R}_{(4)} \longrightarrow \Omega^{-2} \left\{ \mathcal{R}_{(4)} + 6 \left[(\partial(\log \Omega))^2 + \partial^2(\log \Omega) \right] \right\} \quad (\text{A5})$$

It is clear from these equations that in a flat extra dimension S^1/\mathbb{Z}_2 where $y \in [0, \pi]$:

$$\Omega^2 = \frac{M_4^2}{\pi r M_5^3} \quad (\text{A6})$$

will generate the canonical kinetic term for the four-dimensional graviton, where M_4 is the usual 4D Planck scale. In addition it will also generate a canonical kinetic term for the radion:

$$S_4 = \int d^4x \sqrt{-g} \left\{ M_4^2 \mathcal{R}_{(4)} + \frac{1}{2} (\partial\rho)^2 + \dots \right\} \quad (\text{A7})$$

where I have defined the canonical radion field:

$$\rho \equiv \sqrt{12} M_4 \log \left(\frac{r}{r_0} \right) \quad (\text{A8})$$

where r_0 is the classical radius, assumed to be stabilized. Notice that the canonical radion of flat space is the logarithm of R , as opposed to the exponential of R in the warped case [21].

⁵ It is not this simple in general. For example, in the RS model there is also warp factor and more work needs to be done. However, it is not much harder to handle this case.

By letting $r = r_0 + \delta r$ and expanding the logarithm the canonical radion field is related to the usual radion modulus by a constant:

$$\rho = \frac{\sqrt{12}M_4}{r_0}\delta r + \mathcal{O}((\delta r/r_0)^2) \quad (\text{A9})$$

So to compute the radion mass in Planck units ($M_4 \equiv 1$) we compute the radion potential at quadratic order: $V(r) = \frac{1}{2}\mu_r^2(\delta r)^2$. Then $M_{\text{radion}} = \mu_r r_0/\sqrt{12}$.

Using what we now know it is easy to see how the radion can be incorporated into the linearized supergravity action by extending into superspace. To see how this is done notice that the radion modulus appears in a $N = 1$ chiral superfield that contains the \mathbb{Z}_2 -even fifth components of the fields that appear in the 5D supergravity multiplet. This field is called the “radion superfield”:

$$T(x, \theta) = (r + iB_5) + \sqrt{2}\theta\Psi_R^5 + \theta^2 F_T \quad (\text{A10})$$

where B_5 is the fifth component of the graviphoton and Ψ_R^5 is the fifth component of the right-handed gravitino. This is derived in many places such as [22]. Now all we have to do is to include the radion superfield everywhere that it should appear so that we reproduce the correct action in terms of component fields. This was done in [9, 10] for a general class of theories [23]. It is important to notice that this matching must be done before the Weyl rescaling, as explained in [12].

APPENDIX B: KK DECOMPOSITION

In this appendix we will derive the KK decomposition for our model. For simplicity the analysis will only be done for the Φ -sector. It is exactly the same for the Ψ -sector.

To perform the decomposition it is necessary to write out the Lagrangian for the scalar components by integrating out the auxiliary fields from Equation (2). This is given by⁶:

$$\begin{aligned} \mathcal{L} = & \phi^{c\dagger} \left[-\partial^2 + \partial_5^2 - m^2 - \frac{2m}{R}(\delta(y) - \delta(y - \pi)) \right] \phi^c \\ & + \phi^\dagger \left[-\partial^2 + \partial_5^2 - m^2 + \frac{2m}{R}(\delta(y) - \delta(y - \pi)) \right] \phi \\ & - \phi(-\partial_5 + m\Theta(y))S - \phi^\dagger(-\partial_5 + m\Theta(y))S \end{aligned} \quad (\text{B1})$$

where $S = J\delta(y) - J'\delta(y - \pi)$. The extra delta functions in each bracket come from the fact that the mass term is an odd term. This yields the equations of motion:

$$\left[-\partial^2 + \partial_5^2 - m^2 - \frac{2m}{R}(\delta(y) - \delta(y - \pi)) \right] \phi^c = 0 \quad (\text{B2})$$

$$\left[-\partial^2 + \partial_5^2 - m^2 + \frac{2m}{R}(\delta(y) - \delta(y - \pi)) \right] \phi = (-\partial_5 + m\Theta(y))S \quad (\text{B3})$$

⁶ Recall: $\partial_5 \equiv \partial_y/R$

We wish to decompose the fifth dimension so we let:

$$\phi(x, y) = -\frac{J}{2}\Theta(y)e^{-mR|y|} + \sum_{\lambda} \phi_{\lambda}(x)\xi_{\lambda}(y) \quad (\text{B4})$$

$$\phi^c(x, y) = -B(x)e^{+mR|y|} + \sum_{\lambda} \phi_{\lambda}^c(x)\xi_{\lambda}^c(y) \quad (\text{B5})$$

where the first term in ϕ is the particular solution to Equation (B3); it plays the role of a y -dependent vev. This immediately takes care of the source terms for the ϕ field. Notice that this first term is not a zero mode; the coefficient is fixed by the inhomogeneous source terms on the right-hand side of Equation (B3) which eliminate it as a degree of freedom. The even field however does contain a zero mode. We explicitly include a minus sign so that both ϕ and ϕ^c have the same sign in the physical region. This is done purely for convenience and does not change any results.

Now the equations of motion for the KK basis states are:

$$\left[\partial_5^2 - \frac{2m}{R} (\delta(y) - \delta(y - \pi)) \right] \xi_{\lambda}^c = -\lambda^2 \xi_{\lambda}^c \quad (\text{B6})$$

$$\partial_5^2 \xi_{\lambda} = -\lambda^2 \xi_{\lambda} \quad (\text{B7})$$

where we have dropped the delta functions in the equation for ξ_{λ} since it is an odd field and therefore does not feel the delta functions on the boundary. Then $\phi_n(x), \phi_n^c(x)$ are the KK modes with masses $M_{\lambda}^2 = m^2 + \lambda^2$.

The equation for $\xi(y)$ is a very easy equation to solve. Remembering that the odd fields must vanish at the boundaries:

$$\xi_{\lambda}(y) = \sqrt{\frac{2}{\pi}} \sin(ny) \quad \lambda = \frac{n}{R} \quad (\text{B8})$$

The equation for ξ^c is not any more difficult. It is just the Schrodinger equation with delta function potentials and symmetric boundary conditions. We find that $\lambda^2 < 0$ cannot happen so there are no “bound states”. The final solution is:

$$\xi_{\lambda}^c(y) = \sqrt{\frac{2}{\pi}} \sin \left[ny - \tan^{-1} \left(\frac{n}{mR} \right) \right] \quad \lambda = \frac{n}{R} \quad (\text{B9})$$

These are the modes that appear in Equation (19-20). They have been normalized so that $\int_0^{\pi} dy \xi_{\lambda} \xi_{\lambda'} = \delta_{\lambda\lambda'}$. Also notice that the zero mode of ϕ^c is orthogonal to the higher modes, which is easily checked.

APPENDIX C: SUPERGRAVITY CONTRIBUTIONS

In this section, we present the masses and vevs of the hypermultiplets after the lowest-order supergravity effects are taken into account. We make the following definitions:

$$a = \frac{1}{2}m^3\pi^2 J'^2 \quad (C1)$$

$$b = \tilde{\lambda}\pi\mu^2 K^2 (J'/J)^{2\mu/m} \left(\frac{2}{r_0} + \frac{1}{2}\mu\pi\tilde{\lambda} \right) \quad (C2)$$

$$d = (m\mu)^{3/2} JK (J'/J)^{1+\mu/m} \left(\tilde{\lambda} + \frac{2\pi}{\mu r_0} \right) \quad (C3)$$

$$f = \frac{6\pi}{r_0\sqrt{2}} m^{3/2} J' \quad (C4)$$

$$g = \frac{6\pi}{r_0\sqrt{2}} \mu^{3/2} (J'/J)^{\mu/m} \tilde{\lambda} K \quad (C5)$$

These parameters are defined up to terms with $R \neq r_0$. Then in terms of these parameters, the masses, vevs and mixing parameter in the paper are:

$$m_{\tilde{B}}^2 = \frac{2R}{3M_5^3} X_{\tilde{B}} = \frac{r_0}{3M_5^3} \left((a+b) + \sqrt{(a-b)^2 - d^2} \right) \quad (C6)$$

$$m_{\tilde{C}}^2 = \frac{2R}{3M_5^3} X_{\tilde{C}} = \frac{r_0}{3M_5^3} \left((a+b) - \sqrt{(a-b)^2 - d^2} \right) \quad (C7)$$

$$\langle \tilde{B} \rangle = \frac{\alpha}{\sqrt{1+\epsilon^2}} \cdot \frac{f+\epsilon g}{(a+b) + \sqrt{(a-b)^2 - d^2}} \quad (C8)$$

$$\langle \tilde{C} \rangle = \frac{\alpha}{\sqrt{1+\epsilon^2}} \cdot \frac{g-\epsilon f}{(a+b) - \sqrt{(a-b)^2 - d^2}} \quad (C9)$$

$$\epsilon = \frac{b-a}{d} + \sqrt{\left(\frac{b-a}{d} \right)^2 - 1} \quad (C10)$$

where α is the superpotential parameter that cancels the cosmological constant as explained in the paper:

$$\alpha = \sqrt{\frac{1}{u_0} \cdot \frac{\mu K^2}{e^{2\mu\pi r_0} - 1}} \quad (C11)$$

where $U_0 = u_0\alpha^2$ is defined in the text below Equation (28)⁷.

Notice that $m_{\tilde{B}}^2, m_{\tilde{C}}^2 > 0$ for any value of the parameters, so the theory is stable.

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